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Evaluation of empirical formulae for estimation of the longitudinal dispersion in activated sludge reactors

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Abstract

Tracer studies are widely applied to characterize the hydraulic properties of reactors. In the case of activated sludge reactors, however, tracer test results are difficult to interpret due to internal and returned activated sludge recirculation. Empirical formulae can be considered as an alternative method of estimating the hydraulic conditions within the activated sludge reactor. The aim of this study is to evaluate accuracy of four empirical formulae for the full-scale conditions based on the results of tracer studies performed at the Rock Creek Wastewater Treatment Plant (WWTP) in Hillsboro, OR (USA). Values of the dispersion coefficient, E_L, were first estimated using a 1-D advection–dispersion equation and setting a sum of squares of differences between the observed and calculated tracer concentrations to a minimum. The estimated values of E_L coefficient remained within the range of 1043–1580 m^2/h. The best approximation of dispersion was obtained from the formula of Fujie et al. (1983, J. Ferment. Technol. 63(3), 295). Also the formula of Murphy and Boyko (1970, J. San. Eng. ASCE 96(2), 211) generated E_L values of the same order as the optimum E_L. The accuracy of these formulae was further confirmed based on the results of studies reported in the literature.

Keywords: Activated sludge; Dispersion; Empirical formulae; Mixing conditions; Tracer studies

1. Introduction

Many important parameters are influenced by the hydraulic flow characteristics in the activated sludge reactors including organic matter removal and settling properties of the activated sludge (Horan, 1990). The flow patterns in a reactor are described at the extremes as plug flow or completely mixed. A value of the dispersion number (or inverse Peclet number), defined as (E_L/uL), where E_L is the dispersion coefficient [L^{2}T^{-1}], u is the average longitudinal velocity [LT^{-1}], and L is the length scale or length of tank [L], indicates which of the two patterns is approached. When the dispersion number is greater than 0.5–4 (Khudenko and Shpirt, 1986; Murphy and Timpany, 1967; USEPA, 1993), complete mixing can be assumed. On the other hand, long and narrow tanks, for which the dispersion number is smaller than 0.05–0.2 (Khudenko and Shpirt, 1986; Eckenfelder et al., 1985; USEPA, 1993), are considered an approximation of plug flow. In traditional wastewater treatment practice, reactors have generally been designed on the basis of these ideal configurations. However, typical dispersion numbers in wastewater treatment plants (WWTPs) range between 0.1 and 4 (San, 1994), suggesting that deviations from ideal flow have to be taken into consideration. Several complex

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models are available to describe these deviations, of which the tank-in-series model has found widespread application in modeling activated sludge systems. The tank-in-series model reasonably describes only small deviations from complete mixing (Horan, 1990). The degrees of freedom to consider when modeling a particular system are the number of tanks, their respective volumes and internal connections between the tanks (DeClercq et al., 1999). Another alternative for the description of flow conditions is the advection–dispersion equation. Murphy and Timpany (1967) reported that this equation provided a better representation of the response curve than the equal, or non-equal tanks-in-series models when the variance of the curve was used as the criteria of comparison. Some recent studies (Stamou et al., 1999; Makinia and Wells, 2000) have indicated that one-dimensional dispersed flow reactor modeling is appropriate in the case of full-scale activated sludge systems. Alex et al. (2002) demonstrated the application of 3-D computational fluid dynamics (where the model is conceptualized as a number of ideally stirred control volumes with advection and diffusion between those volumes) to describe the behavior of activated sludge tanks in the case of undesirable phenomena such as short circuiting, dead or stagnant zones or sludge settling within the tank.

The experimental technique widely applied to characterize the hydraulic properties of reactors is a tracer test. In the case of activated sludge reactors, however, tracer test results are difficult to interpret due to internal and returned activated sludge (RAS) recirculation (Coen et al., 1998). For example, Petersen et al. (2002) constructed a very complex model of an activated sludge system to fit simulation results to experimental data from a tracer test. This model consisted of the aeration tank (24 tanks-in-series), the channel from the aeration tank to the secondary clarifiers (2 tanks-in-series), an ideal point-settler, a "buffer tank," and the recycle channel from the secondary clarifiers to the aeration tank (5 tanks-in-series).

Without resorting to a tracer test, empirical formulae have been developed to estimate the hydraulic conditions within the activated sludge reactor. These formulae calculate the longitudinal dispersion coefficient, E_L, in terms of operating conditions and physical dimensions of the reactor.

The objective of this paper is to evaluate the accuracy of formulae for estimating E_L from full-scale WWTP conditions based on the results of tracer studies conducted at the Rock Creek WWTP in Hillsboro, OR (USA) and an in-depth study found in the literature (Iida, 1988).

2. Materials and methods

2.1. Estimation of the longitudinal dispersion coefficient, E_L, from tracer studies

2.1.1. Theoretical background

Tracer studies involve finding the age distribution of fluid parcels moving through the reactor. Usually, a tracer is introduced at the reactor inlet, and the tracer concentration is then measured at the outlet as a function of time. The tracer may be introduced instantaneously (an impulse signal) or it may be fed continuously (a step signal). Tracers which can be used for this purpose include fluorescent dyes, radioisotopes, bacteriophages, chemical salts and floats (USGS, 1986; Horan, 1990). The dye tracers have important advantages such as low detection and measurement limits, simplicity, and accuracy in concentration measurements (USGS, 1986). Using tracer studies with an impulse signal, the curves of concentration versus time (or space) can be used to estimate the value of E_L by techniques outlined below:

- Method of moments (French, 1985). By definition, the E_L coefficient (when constant over time) is related to the rate of change of the variance of the tracer cloud:

$$E_{\mathrm{L}} = 0.5\frac{\mathrm{d}(\sigma_x^2)}{\mathrm{d}t}, \tag{1}$$

  where σ_x^2 is the variance of a distribution curve about its mean in space (L^2).

Assuming that the C vs. t curve is approximately Gaussian and that u is approximately constant, Eq. (1) can be transformed to a form where tracer concentrations are measured at a specific point below the point of injection as a function of time:

$$E_{\mathrm{L}} = \frac{u^2}{2}\left[\frac{\sigma_{t_2}^2 - \sigma_{t_1}^2}{\bar{t}_2 - \bar{t}_1}\right], \tag{2}$$

where $\bar{t}_2 - \bar{t}_1$ is the mean times of passage of the tracer cloud past the upstream and downstream sampling points (T), and σ_t^2 the variance of a distribution curve about its mean in time (T^2).

- Relationship between σ_t^2 and E_L using Laplace transforms (Murphy and Timpany, 1967) for a closed system ($E_L(\partial C/\partial x) = 0$ at inlet and outlet) and a constant E_L through the tank:

$$\sigma_t^2 = 2\frac{E_{\mathrm{L}}}{uL} - 2\left(\frac{E_{\mathrm{L}}}{uL}\right)\left[1 - \exp\left(-\frac{uL}{E_{\mathrm{L}}}\right)\right]. \tag{3}$$

  The variance, σ_t^2, for any experimental response curve can be calculated from a dimensionless plot of concentration and time:

$$\sigma_t^2 = \frac{\int_0^\infty \left(\frac{t}{\bar{t}} - 1\right)^2 C \, \mathrm{d}t}{\int_0^\infty C \, \mathrm{d}t}, \tag{4}$$

where $\bar{t}$ is the mean time for the C vs. t distribution.

Combining and rearranging Eqs. (3) and (4), a value of the dispersion coefficient, E_L, can be calculated from field data of C and t.

These two techniques though are not appropriate when sludge recirculation occurs in the activated sludge basin. In order to obtain relevant data for analysis, the RAS recirculation has to be turned off which does not happen under normal operating conditions. The technique that is flexible enough to account for this recirculation is a numerical solution to the 1-D advective dispersion equation without an internal source/sink term. This equation describes transport of the tracer in the activated sludge reactor as follows:

$$\frac{\partial C_k}{\partial t} + \frac{1}{A}\frac{\partial (u\, A\, C_k)}{\partial x} = \frac{1}{A}\frac{\partial}{\partial x}\left(A\, E_L \frac{\partial C_k}{\partial x}\right), \tag{5}$$

where A is the cross-section area of reactor (L^2), C_k the inert tracer concentration (ML^{-3}), E_L the longitudinal dispersion coefficient (L^2T^{-1}), t the time (T), u the bulk velocity along reactor (LT^{-1}), and x the distance along reactor axis (L).

A value of the dispersion coefficient, E_L, can be determined from Eq. (5) using a least-squares fitting approach.

2.1.2. Field test data

The tracer studies were performed at one of the four parallel activated sludge basins at the Rock Creek WWTP located in Hillsboro, Oregon (USA). The basin used in this study had the following dimensions: length 84 m, width 15.6 m, depth 4.9 m (Fig. 1). Based on the design assumptions, the reactor was divided into five equal zones, eventhough the physical baffle existed only between Zone 1A (anoxic zone) and Zone 2 (the first aerobic zone), as shown in Fig. 1. The initial 20% of the reactor volume was used as an anoxic zone in the dry season (May–November), whereas in the wet season (November–April) this zone was aerated with surface aerators. The aeration system in the aerobic zones was equipped with perforated membrane discs submerged 4.2 m below the liquid surface and equally distributed over the bottom area. Air was supplied from one source (blowers) but each zone has a separate outlet pipe. The total flowrate was controlled to maintain continuously a set point of the DO probe located in the middle of Zone 3. The proportions of air supplied to each zone could be adjusted by changing the valve settings.

A series of tracer studies using Rhodamine WT 20% was carried out at the plant to determine the magnitude of dispersion in the activated sludge reactor and estimate the value of E_L coefficient. The samples were analyzed in a Turner model 112 fluorometer with a general-purpose UV lamp. Before the studies, the fluorometer was calibrated, and a batch test was performed to exclude possible adsorption of the dye by activated sludge flocs.

Each tracer test started with stabilizing the wastewater flowrate and air supply to the reactor (Table 1). The RAS flowrate was set at 40% of the wastewater flowrate and the internal recirculation of the mixed liquor was turned off. Samples of the mixed liquor for a background fluorescence analysis were taken from the outlet of the reactor and from an identical neighboring reactor. Then 0.5 dm^3 of the Rhodamine was injected at the reactor inlet. A sampling point was established at the reactor outlet. Samples of volume 0.15 dm^3 were taken at the reactor outlet and analyzed by the fluorometer accounting for a temperature correction of the fluorometer readings.

In order to estimate the impact of the return activated sludge on the distribution curves, two additional tests were performed. In both cases, the same amount of the dye (i.e., 0.5 dm^3) was injected at the reactor inlet (Test 4) and below the original sampling point (Test 5). Samples were taken at the original sampling point (Test 4) and at the inlet of the return activated sludge to the reactor (Test 5).

Besides this study, Iida (1988) performed tracer studies with lithium chloride to evaluate mixing conditions in six full-scale aeration basins. In the tanks

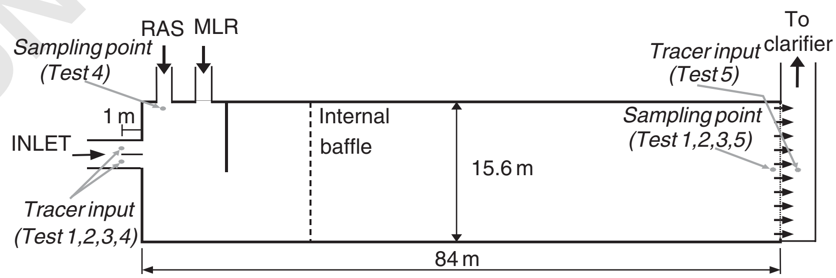

Fig. 1. Sketch showing locations of the tracer input and the sampling point (not drawn to scale) during the tracer studies in the activated sludge basin at the Rock Creek WWTP.

Table 1
Wastewater flowrate and air flowrate to the reactor during the tracer studies carried out at the Rock Creek WWTP

Test	Wastewater flowrate (m^3/h)			Air flowrate (m^3/h)		
	Max.	Min.	Average	Max.	Min.	Average
Test 1	2399	2308	2342	5894	4572	4995
Test 2	1967	1793	1888	5992	5911	5952
Test 3	2661	2509	2598	6096	5958	6064
Test 4	2599	2213	2290	6021	4682	4914
Test 5	1972	1498	1754	7849	6658	7095

Table 2
Summary of the reactor characteristics evaluated in study from Iida (1988)

Reactor	Volume (V) (m^3)	Length (L) (m)	Width (W) (m)	Depth (H) (m)	Flowrate (Q) (m^3/h)	Air flow (Q_{air}) (m^3/min)	E_L (m^2/h)
1	3310	100.2	7.5	4.4	360	3990	597–826
2	3502	52.4	8.3	8.1	732	960	208–327
3	3502	52.4	8.3	8.1	732	930	524–792
4	3565	113.3	7.2	4.4	654	2046	342–371
5	1782.5	113.3	7.2	4.4	612	2220	340–436

studied, coarse porous plate diffusers were set along one side of the tanks or were set in rows longitudinally. Some basins revealed a relatively constant dispersion throughout the entire length of the reactor. These experimental data were selected for evaluating the accuracy of empirical formulae for E_L described in the next section. The reactor characteristics, including dimensions, flowrates and measured dispersion coefficients, are presented in Table 2.

2.1.3. Model development

The advection–dispersion model in Eq. (5) described the advective and dispersive transport of a conservative tracer. The equation was solved using an explicit finite difference numerical scheme. To balance numerical accuracy with computational time, the activated sludge reactor at the Rock Creek WWTP was divided into 21 model segments as presented in Appendix A. The reactor was subjected to the following initial and boundary conditions:

- Initial conditions: for model segment $i = 2$, $C_2 = M_{tracer}/V_2$, and for all segments $i \neq 2$, $C_i = 0$, where M_{tracer} is the mass of tracer added as a function of time, V_2 is the volume of segment 2, C_i is the concentration of model segment i.
- Boundary conditions: $E_L(\partial C/\partial x) = 0$ at all reactor walls and exit.

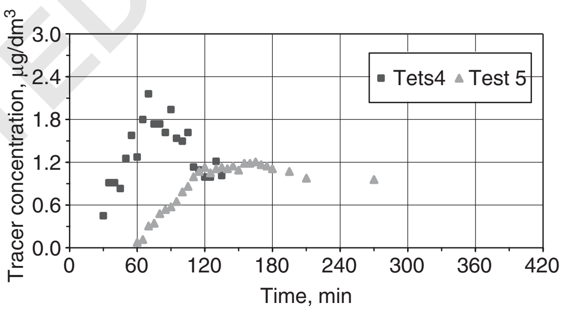

Fig. 2. Results of additional tracer studies (Tests 4–5) for estimating the impact of RAS on the tracer concentration profile at the reactor outlet.

Dispersion in the secondary clarifier and the connecting pipes was not measured. Therefore, the tracer concentrations entering the reactor with RAS were approximated based on the measurements of Test 4 (Fig. 2). From this model, values of E_L were estimated by minimizing the error of predicted concentrations as a function of time. In addition, the distribution of tracer concentrations vs. time was estimated with the tanks-in-series model (Makinia and Wells, 2000). The response curve to a pulse input for N equal completely mixed tanks-in-series is given by the following expression:

$$\frac{C_e}{C_0} = \frac{N^N}{(N-1)!}\left(\frac{t}{\bar{t}}\right)^{N-1} \exp\left(\frac{-N\,t}{\bar{t}}\right), \quad (6)$$

where N is the number of reactors, C_e is the exit concentration, C_0 is the initial concentration. This equation does not have a dispersion coefficient and each part of the aeration basin is treated as fully mixed.

For comparison, the E_L values were also determined using various empirical formulae outlined below.

2.2. Estimation of the longitudinal dispersion coefficient, E_L, from empirical formulae

Murphy and Boyko (1970): Tanks of different width to depth ratios ranging from 0.87 to 2.04 were studied. Various combinations of width and depth were selected tentatively as the "characteristic length" and the most successful correlation ($r^2 = 0.885$ for 96 different test conditions) was obtained when the tank width was selected. Consequently, the following correlation relating the longitudinal dispersion coefficient, the tank width, and specific air flow rate per unit tank volume was proposed:

$$\frac{E_L}{W^2} = 3.118(q_A)^{0.346}, \tag{7}$$

where q_A is the air flowrate per unit reactor volume (T^{-1}), and W the reactor width (L).

It should be noted that USEPA (1993) recommended the use of Eq. (7) as an acceptable approximation of the dispersion coefficient in reactors with both fine and coarse bubble diffused air systems.

Harremoes (1979): The longitudinal dispersion coefficient was determined by the general flow pattern of the reactor as generated by the air supply. The tanks studied included a bench-scale reactor with diffuser stones on the side and at the bottom of tanks, two full-scale aeration tanks in Denmark and literature data on full-scale systems. It was assumed that the coefficient was primarily a function of the characteristic velocity, defined as $(g\,q'_A)^{1/3}$ where g is the gravity acceleration [LT^{-2}] and q'_A is air flowrate per unit length of reactor [L^2T^{-1}] with corrections for the geometry of the tank. Multi-parameter regression analysis performed for the results of two pilot plant studies (45 measurements) and one full-scale study (26 measurements) gave the following result:

$$\frac{E_L}{(gq'_A)^{1/3}\,W} = 2.4 \times 10^{-3}\left(\frac{H}{W}\right)^{-0.68} Re_g^{0.26}. \tag{8}$$

The Reynolds number associated with the aeration intensity (Re_g) was defined as

$$Re_g = \frac{(gq'_A)^{1/3}\,H}{\nu_l}, \tag{9}$$

where H is the reactor depth (L), ν_l the kinematic viscosity of liquid (LT^{-2}).

Fujie et al. (1983): Two full-scale tanks with diffusers (porous plastic tubes or porous ceramic plates) located near the bottom of one side of the tanks were studied. The longitudinal dispersion coefficient was related to spiral liquid circulation by applying random walk theory. The relationship between E_L and the variance of the displacement of a liquid element, σ_x^2, was given by Eq. (1). The variance σ_x^2 was assumed to be proportional to the displacement of the liquid element ($\pm\lambda(H+W)$) and was calculated from the following empirical relationship:

$$\sigma_x^2 = N_c[\lambda(H+W)]^2, \tag{10}$$

where N_c is the number of circulations in the vertical cross-section, λ the non-dimensional correction factor which makes $\lambda(H+W)$ the displacement from the average flow during one circulation.

The time t_N required for a liquid element of interest to circulate N_c times in the vertical cross section was expressed as

$$t_N = \frac{2N_c\,\xi\,(H+W)}{\xi' u_{ls}}, \tag{11}$$

where u_{ls} is the spiral circulation rate (LT^{-1}), ξ the non-dimensional correction factor which makes $\xi(H+W)$ the average traveling distance of the liquid element in the vertical cross-section, ξ' the non-dimensional correction factor which makes $\xi' u_{ls}$ the average spiral circulation rate at liquid surface.

Rearranging Eq. (1) by substituting σ_x^2 and t_N from Eqs. (10) and (11), respectively, gives

$$E_L = \lambda'^2 u_{ls}\,(H+W), \tag{12}$$

where

$$\lambda'^2 = \frac{\lambda\,\xi'}{4\xi}. \tag{13}$$

The following equations for λ'^2 and u_{ls} were developed based on literature data and own studies of the authors:

$$\lambda'^2 = 0.0115\left(1+\frac{H}{L}\right)^{-3} u_g^{-0.34}, \tag{14}$$

$$u_{ls} = a_d\left[h\,u_g\left(\frac{h}{H}\right)^{1/2}\left(\frac{H}{W}\right)^{1/3}\right]^{m_d}, \tag{15}$$

where a_d, m_d are the empirical constants dependent on the type of air diffuser, h the diffuser depth (L), t_N the time required for a liquid element of interest to circulate N times in the vertical cross section (T), and u_g the superficial gas velocity (LT^{-1}) (Table 3).

The final formula for E_L was obtained by rearranging Eq. (12) with λ'^2 and u_{ls} from Eqs. (14) and (15), and presented in the following form:

Table 3
Values of parameters a_d and m_d in Eqs. (14) and (15), where $\Phi = hu_g(h/H)^{1/2}(H/W)^{1/3}$

Type of air diffuser	Φ (cm²/s)	m_d	a_d
Fine bubble types[a]	$\Phi \leqslant 20$	0.64	7.0
	$\Phi > 20$	0.46	12.0
Coarse bubble types[b]	$\Phi \leqslant 20$	0.78	3.5
	$\Phi > 20$	0.56	4.9

[a]Porous plates and tubes.
[b]Perforated plates and tubes, single nozzles and others.

$$E_L = 0.0115\left(1+\frac{H}{L}\right)^{-3} u_g^{-0.34} \times a_d\left[hu_g\left(\frac{h}{H}\right)^{1/2}\left(\frac{H}{W}\right)^{1/3}\right]^{m_d}(H+W). \quad (16)$$

Khudenko and Shpirt (1986): Various bench-scale and full-scale tanks with diffused air systems, both fine and coarse porous plates, were studied. The width of the aerator band ranged from 12.5% to 100% of the tank width. The dispersion coefficient was related to geometric and dynamic parameters through a general relationship:

$$E_L = f_1(H, W, L, w, u_g, u, v_l). \quad (17)$$

Using the Buckingham π-theorem, the following dimensionless equation, composed from the parameters listed in Eq. (17), was derived:

$$\frac{E_L}{uL} = A_1\, Re_g^{\alpha_1}\, Re_l^{\alpha_2}\left(\frac{L}{W}\right)^{\alpha_3}\left(\frac{H}{W}\right)^{\alpha_4}\left(\frac{w}{W}\right)^{\alpha_5}, \quad (18)$$

where w is the width of aeration band (L), A_1 the empirical constant, and $\alpha_1 \ldots \alpha_5$ the empirical constants.

Reynolds numbers associated with the aeration intensity (Re_g) and the fluid flow in reactor (Re_l) were defined as

$$Re_g = \frac{u_g H}{v_l}, \quad (19)$$

$$Re_l = \frac{uH}{v_l}. \quad (20)$$

Values of the dispersion coefficient were found using Eq. (3) and correlated with the hydrodynamic parameters of the reactor. The final form of Eq. (18) was found to be

$$\frac{E_L}{uL} = 4.2\, Re_g^{0.60}\, Re_l^{-0.75}\left(\frac{L}{W}\right)^{-0.9}\left(\frac{H}{W}\right)^{0.80}\left(\frac{w}{W}\right)^{0.28}. \quad (21)$$

Eq. (21) suggested that the longitudinal dispersion of flow in aeration tanks increased with an increase in the aeration intensity (or Re_g), the tank depth, and the aeration band width. Longitudinal dispersion decreased with the increase in the velocity of flow along the reactor (or Re_l), and the tank length. The width of the tank produced only a slight effect on the mixing pattern.

Chambers and Jones (1988): Measurements of the dispersion number, performed by tracer tests in 24 full-scale WWTPs, revealed that the value of the dispersion number could be considered virtually constant in diffused-air systems and had a value approximately equal to 0.068 m²/s (245 m²/h) with the accuracy ±15% for the following conditions: $2\,\text{m} < W < 20\,\text{m}$, $2.4\,\text{m} < \cap$ $H < 6\,\text{m}$, $28\,\text{m} < L < 500\,\text{m}$, $0.7 < r < 1.5$, $1.3\,\text{h} < t < 8\,\text{h}$ (where r is the RAS recirculation ratio and t is the hydraulic retention time).

3. Results and discussion

The results of three tracer tests carried out in the activated sludge reactor at the Rock Creek WWTP are presented in Fig. 3. The same figure also illustrates numerical simulations of these tests using different models. In order to determine the value of E_L coefficient in the advection–dispersion model, a sum of squares of differences between the observed and predicted tracer concentrations were set to a minimum. The calculated values of E_L ranged from 1043 to1580 m²/h (1130 m²/h for Test 1, 1580 m²/h for Test 2, and 1043 m²/h for Test 3). Based on the operational data listed in Table 1, it is apparent that the E_L values were more related to the wastewater flowrate ($\sim 1/Q$) than to the air flowrate ($\sim Q_{air}$). Without accounting for the tracer recirculated with RAS, the estimated E_L values were higher by 7–9% compared to the case considering the impact of recirculated tracer. The arrival of peak concentrations was not considerably affected by the recirculated tracer, but its impact on the distribution curve increased over the time of the test. The comparison of model predictions with and without the tracer in RAS allowed one to estimate the time after which the contribution of the recirculated tracer became greater than the tracer injected at the beginning of the test. This time was approximately 160 min (Test 1), 235 min (Test 2) and 180 min (Test 3). The additional test results, presented in Fig. 2, revealed that the tracer was detected in the RAS entering the reactor after less than 0.5 h and that the maximum concentration reached 2.2 μg/dm³. After injecting directly below the original sampling point (reactor outlet), the tracer was detected at the outlet from the reactor after less than 1 h and the maximum concentration reached 1.2 μg/dm³.

The curves obtained using the tanks-in-series model (Eq. (6)) did not adequately reflect the actual flow pattern in the reactor (Fig. 3), although the reactor had

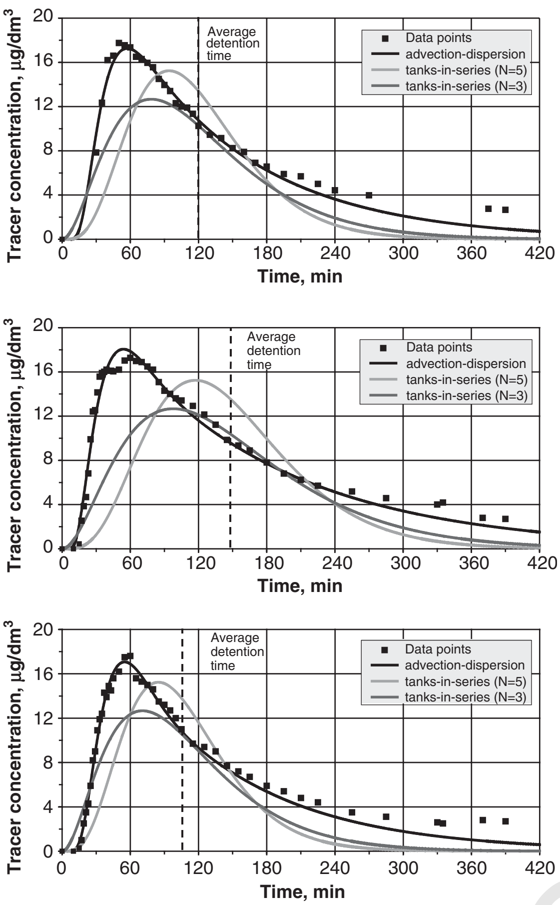

Fig. 3. Observed tracer concentrations at the reactor outlet and numerical simulation of Tests 1–3.

been designed and operated as a series of five completely mixed zones of equal size. Under such an assumption, the predicted peak concentrations were lower by approximately 12–17% and delayed by approximately 30–60 min in comparison to the actual peaks. The tanks-in-series model with a smaller number of tanks also did not predict accurately the distribution of tracer concentrations. For comparison, the calculated concentrations with the 3 tanks-in-series model are also presented in Fig. 3.

Values of the dispersion coefficient were also estimated from the empirical formulae (Table 4). It should be noted that some of them were derived for aerators installed on one side of the tank. Such aerators system can generate a relatively small longitudinal dispersion by ''water rolls'' in contrary to aerators distributed equally over the tank bottom which have only local rolls and thus generate high dispersion. The best approximation of dispersion was obtained from the formula of Fujie et al. (1983) followed by the formula of Murphy and

Table 4
The dispersion coefficient, E_L, calculated from various empirical formulae using the input data from Tests 1–3

Method/formula	E_L (Test1) (m^2/h)	E_L (Test2) (m^2/h)	E_L (Test3) (m^2/h)	Reference
Tracer studies using 1-D advective dispersion model to estimate E_L	1130	1580	1043	
$E_L = 3.118(q_A)^{0.346} W^2$	1841	1956	1969	Murphy and Boyko, 1970
$E_L = 2.4 \times 10^{-3}(H/W)^{-0.68} Re_g^{0.26}(gq_A')^{1/3} W$	7334	7894	7956	Harremoes, 1979
$E_L = 0.0115\left(1+\frac{H}{L}\right)^{-3} u_g^{-0.34} a_d \left[hu_g\left(\frac{h}{H}\right)^{1/2}\left(\frac{H}{W}\right)^{1/3}\right]^{m_d}(H+W)$	993	1014	1016	Fujie et al., 1983
$E_L = 4.2\, Re_g^{0.60} Re_l^{-0.75}\left(\frac{L}{W}\right)^{-0.9}\left(\frac{H}{W}\right)^{0.80}\left(\frac{w}{W}\right)^{0.28} uL$	59	63	68	Khudenko and Shpirt, 1986

Boyko (1970). The calculated values of E_L from both formulae were used in the advection–dispersion formula to predict the effluent tracer concentration for the data of Test 2 (Fig. 4). A measure of the prediction accuracy was the average relative deviation (ARD), defined as

$$\mathrm{ARD} = \frac{1}{n}\sum_{i=1}^{N}\frac{\left|(x_i - y_i)\right|}{x_i} \times 100\%, \qquad (22)$$

where ARD is the average relative deviation (%), n the number of experimental data points, x_i the measured tracer concentrations, and y_i the predicted tracer concentrations.

The values of ARD for both formulae were 19.7% and 34.3%, respectively, whereas the minimum ARD was 17.6% for an E_L equal to 1580 m^2/h. The other formulae, evaluated in this study, generated results substantially different from the E_L values calculated based on the data from tracer studies. Moreover, the actual values of E_L were also much higher in comparison with the average value of E_L (245 m^2/h) estimated by Chambers and Jones (1988) at 24 activated sludge reactors with similar dimensions and operating conditions to those at the Rock Creek WWTP.

The accuracy of empirical formulae was confirmed based on the literature data of Iida (1988). The calculated values of E_L versus the reported values (estimated from the tracer studies and the advection–-dispersion model) are presented in Fig. 5. The formula of Khudenko and Shpirt (1986) was not applicable to the type of reactors under study since coarse porous plate diffusers were set along one side of the tanks, and hence, it was not possible to estimate the value of the w coefficient (width of aeration band). In the case of the formula of Harremoes (1979), the accuracy improved significantly when Reynolds number, Re_g, defined in Eq. (9) was replaced by Re_g calculated from Eq. (20).

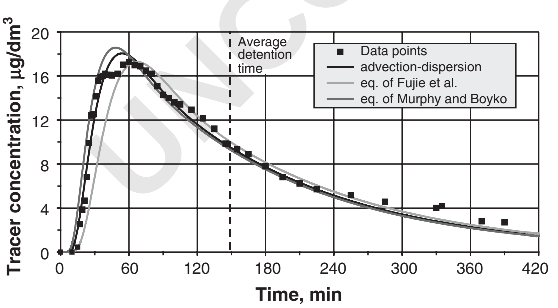

Fig. 4. Comparison of the accuracy of the advection–dispersion equation for the data from Test 2 using the EL values estimated from tracer studies and empirical formulae of Fujie et al. (1983) and Murphy and Boyko (1970).

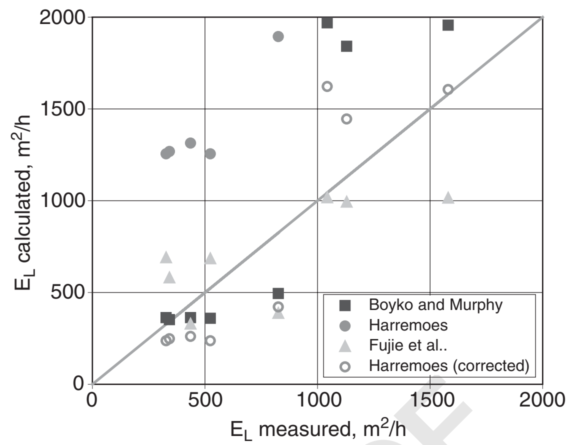

Fig. 5. Comparison of the measured and calculated values of EL coefficient in the studies of Iida (1988) and the current study.

4. Conclusions

In this study, several empirical formulae for calculating the E_L coefficient were compared to the values of this coefficient estimated from the tracer studies by minimizing the prediction error in the 1-D advection–dispersion equation. Some of the evaluated formulae confirmed their capability to approximate mixing conditions in the full-scale activated sludge reactor. A principal limitation of these empirical formulae is that they are applicable only to the aerated zones since the calculated E_L coefficient is related to the aeration intensity in the reactor. The best accuracy in comparison to the results of three tracer studies was obtained for the formula of Fujie et al. (1983). When the calculated E_L coefficients were applied to the advection–dispersion, the ARD values were higher only by less than 2.1% from the ARD corresponding to the optimum value of E_L. The values of the same order as the optimum E_L were also generated by the formula of Murphy and Boyko (1970), but the difference between ARDs reached 16.7% (Test 2). The prediction capabilities of these two formulae was further confirmed based on the results of tracer studies reported in the literature. The accuracy of the formula of Harremoes (1979) improved significantly when the Reynolds number, Re_g, defined in Eq. (9), was replaced by Re_g calculated from Eq. (19). The corrected formula of Harremoes (1979) generated the E_L values similar to those obtained from the two formulae mentioned above.

5. Uncited reference

Koch et al., 2000.

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Appendix A

The activated sludge basin at the Rock Creek AWWTP was divided into 21 cells according to the scheme presented in Fig. 6. The concentrations of the tracer in each cell were calculated based on the mass balance including the advective and dispersive terms.

Cell 1:

$$V_1 \frac{C_1^{n+1} - C_1^n}{\Delta t} = A_1 E_{\mathrm{L}} \frac{C_2^n - C_1^n}{\Delta x_1} - u_1 A_1 C_1^n + Q_{\mathrm{RAS}} C_{\mathrm{RAS}}, \tag{A.1}$$

$$C_1^{n+1} = C_1^n \left(1 - \frac{u_1\,\Delta t}{\Delta x_1} - \frac{E_{\mathrm{L}}\,\Delta t}{\Delta x_1^2}\right) + C_2^n \frac{E_{\mathrm{L}} \Delta t}{\Delta x_1^2} + \frac{\Delta t}{V_1} Q_{\mathrm{RAS}} C_{\mathrm{RAS}}, \tag{A.2}$$

where

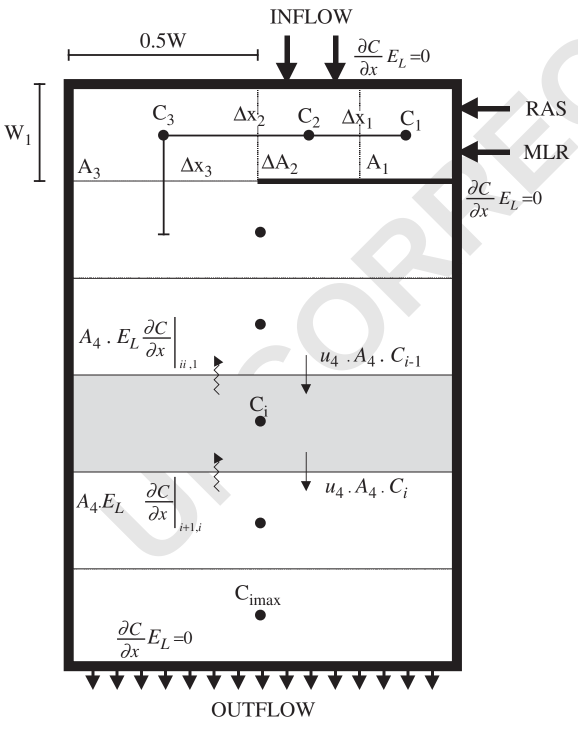

Fig. 6. Scheme of the activated sludge basin at the Rock Creek WWTP for numerical calculations.

$$u_1 = \frac{Q_{\mathrm{RAS}} + Q_{\mathrm{MLR}}}{A_1}. \tag{A.3}$$

Cell 2:

$$V_2 \frac{C_2^{n+1} - C_2^n}{\Delta t} = A_2 E_{\mathrm{L}} \frac{C_3^n - C_2^n}{\Delta x_2} - A_1 E_{\mathrm{L}} \frac{C_2^n - C_1^n}{\Delta x_1} + u_1 A_1 C_1^n - u_2 A_2 C_2^n, \tag{A.4}$$

$$C_2^{n+1} = C_2^n \left[1 - \frac{u_2\,\Delta t}{\Delta x_1} - \frac{E_{\mathrm{L}}\,\Delta t}{\Delta x_1}\left(\frac{1}{\Delta x_1} + \frac{1}{\Delta x_2}\right)\right] + C_1^n \left(\frac{u_1\,\Delta t}{\Delta x} + \frac{E_{\mathrm{L}}\,\Delta t}{\Delta x_1^2}\right) + C_3^n \frac{E_{\mathrm{L}}\,\Delta t}{\Delta x_1 \Delta x_2}, \tag{A.5}$$

where

$$u_2 = \frac{Q + Q_{\mathrm{RAS}} + Q_{\mathrm{MLR}}}{A_2}. \tag{A.6}$$

Cell 3:

$$V_3 \frac{C_3^{n+1} - C_3^n}{\Delta t} = A_3 E_{\mathrm{L}} \frac{C_4^n - C_3^n}{\Delta x_3} - A_2 E_{\mathrm{L}} \frac{C_3^n - C_2^n}{\Delta x_2} + u_2 A_2 C_2^n - u_3 A_3 C_3^n, \tag{A.7}$$

$$C_3^{n+1} = C_3^n \left(1 - \frac{u_3\,\Delta t}{W_1} - \frac{E_{\mathrm{L}}\,\Delta t}{W_1\,\Delta x_3} - \frac{E_{\mathrm{L}}\,\Delta t}{0.5B\,\Delta x_2}\right) + C_2^n \left(\frac{u_2\,\Delta t}{0.5B} + \frac{E_{\mathrm{L}}\,\Delta t}{0.5B\,\Delta x_2}\right) + C_4^n \frac{E_{\mathrm{L}}\,\Delta t}{W_1\,\Delta x_3}, \tag{A.8}$$

where

$$u_3 = \frac{Q + Q_{\mathrm{RAS}} + Q_{\mathrm{MLR}}}{A_3}. \tag{A.9}$$

Cell *i*:

$$V_4 \frac{C_i^{n+1} - C_i^n}{\Delta t} = A_4 E_{\mathrm{L}} \frac{C_{i+1}^n - C_i^n}{\Delta x_4} - A_4 E_{\mathrm{L}} \frac{C_i^n - C_{i-1}^n}{\Delta x_4} + u_4 A_4 C_{i-1}^n - u_4 A_4 C_i^n, \tag{A.10}$$

$$C_i^{n+1} = C_i^n \left(1 - \frac{u_4\,\Delta t}{\Delta x_4} - 2\frac{E_{\mathrm{L}}\,\Delta t}{\Delta x_4^2}\right) + C_{i-1}^n \left(\frac{u_4\,\Delta t}{\Delta x_4} + \frac{E_{\mathrm{L}}\,\Delta t}{\Delta x_4^2}\right) + C_{i+1}^n \frac{E_{\mathrm{L}}\,\Delta t}{\Delta x_4^2}. \tag{A.11}$$

Cell $i_{\max}$:

$$V_4 \frac{C_{i\max}^{n+1} - C_{i\max}^n}{\Delta t} = -A_4 E_{\mathrm{L}} \frac{C_{i\max}^n - C_{i\max-1}^n}{\Delta x_4} + u_4 A_4 C_{i\max-1}^n - u_4 A_4 C_{i\max}^n,$$

$$C_{i\max}^{n+1} = C_{i\max}^n \left(1 - \frac{u_4\,\Delta t}{\Delta x_4} - \frac{E_{\mathrm{L}}\,\Delta t}{\Delta x_4^2}\right) + C_{i\max-1}^n \left(\frac{u_4\,\Delta t}{\Delta x_4} + \frac{E_{\mathrm{L}}\,\Delta t}{\Delta x_4^2}\right). \tag{A.13}$$

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