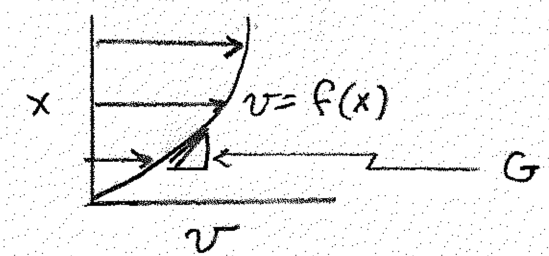
Effective mixing depends on TWO variables: G & t_d

1. $G = \frac{\partial v}{\partial x}$ = GRADIENT OF VELOCITY

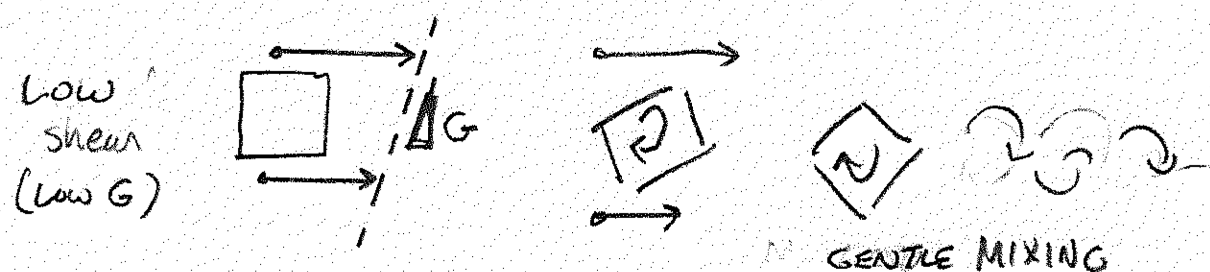


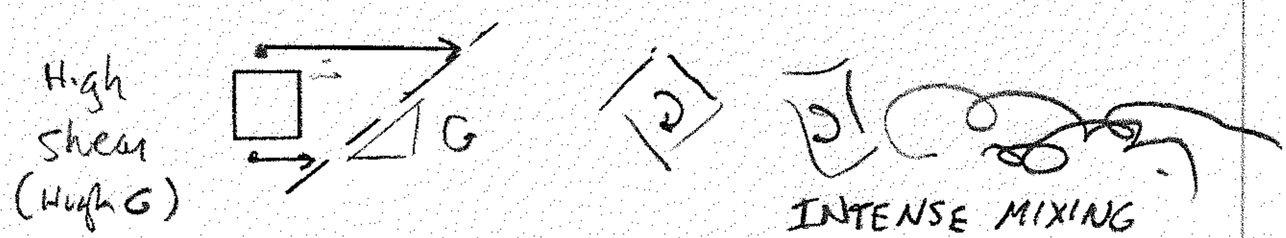
Why? Newton's Law of Viscosity

$$\tau_x = \mu \frac{\partial v}{\partial x}$$

Shear stress (τ_x) is proportional to G ($\frac{\partial v}{\partial x}$)
(where viscosity μ is proportionality factor)

Fluid Shear induces turbulent mixing: ROTATION





∴ - G dictates INTENSITY OF MIXING

And - Fluid mixing requires energy input; i.e., POWER $\left(\frac{J}{s}\right) = W$

So, since Mixing Intensity proportional to mech. energy input
Mixing Intensity = G, proportional to mech. energy input = Power Input (Use)

(2) The cumulative EFFECT of mixing depends <u>not</u> just on <u>intensity</u> but also <u>DURATION</u> of mixing t_d

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LONG MIXING TIME : More complete mix
SHORT MIXING TIME : Less complete mix
] FOR A GIVEN G

ANALOGOUS TO C·t concept in DISINFECTION

- C: Intensity of disinfection
- t: Duration of disinfection

∴ OVERALL : Mixing Effectiveness governed by the product

$$\boxed{G \cdot t_d}$$

<u>Note</u> that, conveniently this is <u>DIMENSIONLESS</u> number

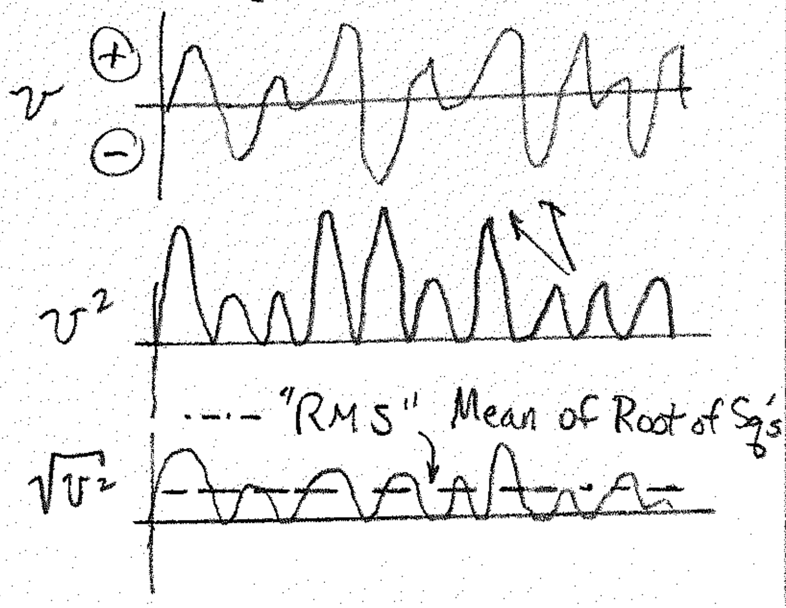
UNITS: $G = \frac{v}{L} = \frac{L/T}{L} = T^{-1}$

$t_d = T$

$\therefore G \cdot t_d = T^{-1} \cdot T$

<u>Note Also</u>: Shear-induced mixing depends on MAGNITUDE of velocity gradient (both ⊕ & ⊖ directions induce shear & mixing.

IMAGINE velocities fluctuating at pt. →



G ≡ RMS velocity
Root Mean Square

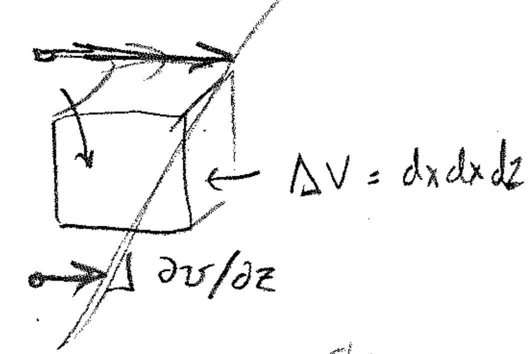
(Gives mean of absolute values of G)

FORCE & POWER BALANCE: Connect P to G ⊛

$P \equiv \frac{\text{WORK}}{\text{TIME}} = \frac{\text{Force} \cdot \text{Dist}}{\text{time}}$

$P = F \cdot v$

INFINITESIMAL CONTROL VOLUME



$\tau = \frac{F}{A}$ ← Shear Force

$\Delta P = F \Delta v$ ← change in Power (M.E. Content) = Shear forces applied times change in velocity

$\therefore \; F_\tau = \tau \, dA = \tau \, dx \, dy$

$\Delta v = \frac{\partial v}{\partial z} dz$

$\Delta P = F \Delta v = \tau \frac{\partial v}{\partial z} dx \, dy \, dz$

$\tau \frac{\partial v}{\partial z} \Delta V$

$\frac{\text{POWER INPUT}}{\text{VOLUME}} \quad \frac{\Delta P}{\Delta V} = \tau \frac{\partial v}{\partial z} = \mu \frac{\partial v}{\partial z} \cdot \frac{\partial v}{\partial z}$

$$\frac{P}{V} = \mu \left(\frac{\partial v}{\partial z} \right)^2 = \mu G^2$$

OR

$$G = \left(\frac{P}{\mu V} \right)^{\frac{1}{2}}$$

⊛ NOTE: This is not essential for this class but provided for completeness and those interested in fluid mechanics

DESIGN PROCESS:

(1) Find desired $G \cdot t_2$ (say 27,000)

(2) Pick reasonable paire (E.g. Table 13.3)

a) Use G to solve for P

b) Use t_d to get mixer volume

$$V = Q t_d$$

(3) Use V to find DIAMETER D

(4) Use Table 13.4 to get ϕ, power coeff.

AND $$\phi = \frac{P}{\rho N^3 D^5}$$

(5) Solve for N (s^{-1})

(6) Convert to R.P.S. $= \frac{N}{\pi}$