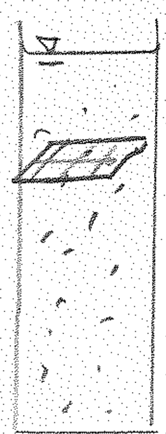
**Question**: If all particles have some finite vertical settling velocity (due to net gravity) won't they eventually settle out, no matter how small.

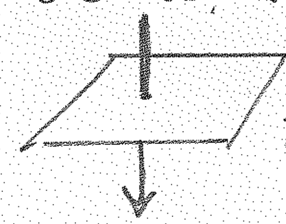
**Answer**: NO. But reason is a bit tricky (but interesting)



Consider uniform suspension (well mixed) in a settling column (left.)

← Consider particle flux at any depth

Advective flux downward due to net gravity force



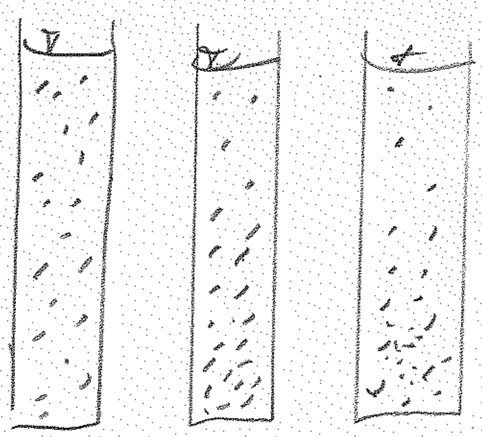
FLUX ≡ Mass per unit time per unit AREA

$$J_s = v_s C$$

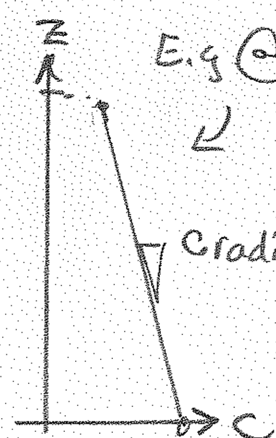
Adv Flux = (Settling vel.)(Particle Conc.)

E.g.: $\mu g\, cm^{-1} s^{-1}$    $cm/s$    $\mu g/cm^3$ (= mg/L)

This flux causes the top of column to have lower C and particles build up near bottom, raising C
CREATES VERTICAL GRADIENT

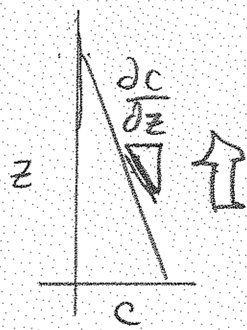


$t_0$    $t_1$    $t_2$



<u>But</u>, even in a stagnant (quiescent) system, Brownian motion ensures random particle movement in all directions. Any turbulence just greatly enhances this.

GRADIENT + RANDOM MOTIONS = DIFFUSION ("DOWN" THE GRAD)



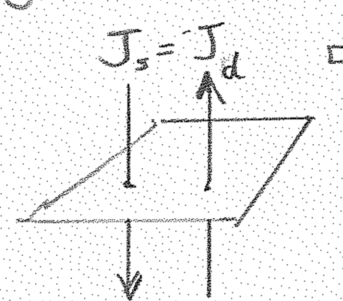

DIFFUSIVE FLUX OF PARTICLES UPWARD

$$J_d = -D_p \frac{\partial c}{\partial z}$$

<u>Fick's Law</u>

Diffusivity of Particles (due to Brownian motion; typically smaller than molecular diffusivity)

Hence settling builds up a gradient until the diffusive upward flux equals the settling flux. A <u>steady state</u> ensues with <u>no further net settling</u>.




$$\Rightarrow \quad v_s C = -D_p \frac{\partial c}{\partial z}$$

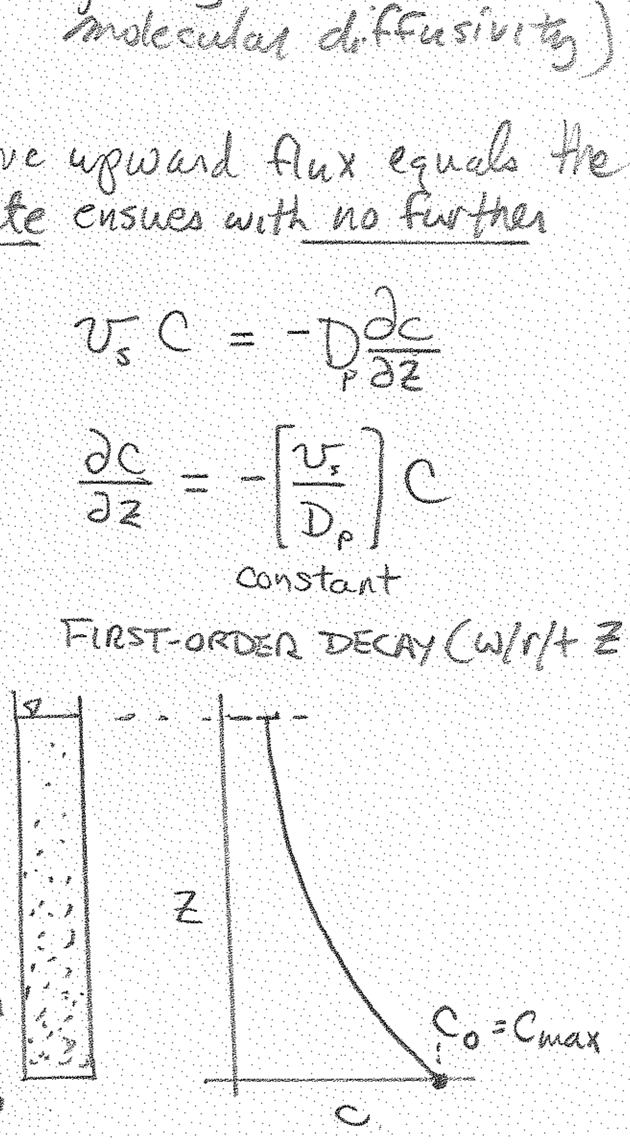
$$\frac{\partial c}{\partial z} = -\underbrace{\left[\frac{v_s}{D_p}\right]}_{\text{constant}} C$$

FIRST-ORDER DECAY (w/r/t Z)

S.S. Soln:

$$\boxed{C(z) = C_0 \, e^{-\left(\frac{v_s}{D_p}\right) z}}$$

∴ Particle concentration is max at bottom $C_0 = C_{max}$ and a STEADY (unchanging) exponentially decaying distrib'n of C with distance Z from bottom



<u>MAGNITUDE</u>? For some particle-size ranges a significant "curve" in $C(z)$ sets up but for most particles subject to Brownian motion (i.e. pretty small) the "curvature" of the vertical distribution is slight to ~ nonexistent.

<u>EXAMPLES</u> For simplicity, consider two monodisperse (uniform) suspensions of clay ~ bracketing clay sizes

| | SIZE | STOKES VELOCITY |
|---|---|---|
| LARGE CLAY | $d_1 = 1\ \mu m$ | $7 \times 10^{-5}$ cm/s |
| SMALL CLAY | $d_2 = 0.01\ \mu m$ | $5 \times 10^{-7}$ cm/s |

First, notice time to settle just <u>1 cm</u>: (Ignore diffusion)

LARGE 14,290 s = 4 hr

SMALL $2 \times 10^{6}$ s = 23 days

Now add in diffusive (Brownian) flux

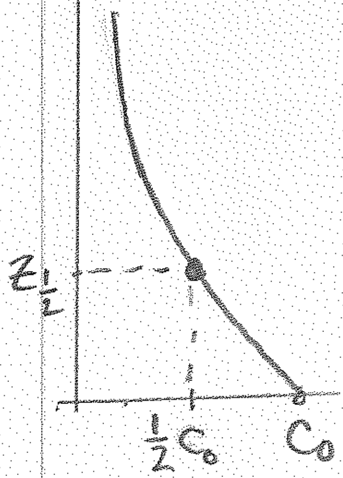
$$D_{p1} = 5 \times 10^{-9}\ cm^2/s$$
$$D_{p2} = 5 \times 10^{-7}\ cm/s$$

FOR COMPARISON
$D_{mol} \approx 10^{-5}\ cm^2/s$
DISSOLVED SOLUTES

Easy index of "curve" or "drop off" in $C(z)$ is spatial "half-life"

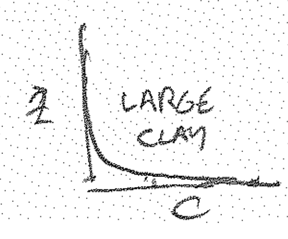
$z$ where $C = \frac{1}{2} C_0$

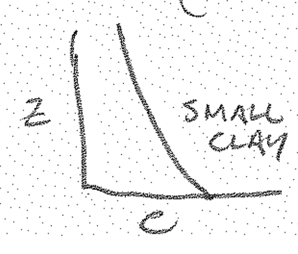
$$z_{\frac{1}{2}} = \frac{\ln 0.5}{-(v_s/D_p)} = \frac{0.69}{(v_s/D_p)}$$




<u>LARGE</u>: $z_{\frac{1}{2}}$ JUST <u>5 μm</u> (Really piles up before SS.)

<u>SMALL</u> $z_{\frac{1}{2}}$ 50 cm (0.5 m)



BUT Let's introduce just the tiniest bit of natural turbulence and mixing
(Say, a VERY, VERY gentle flow through a clarifier)

Now $D^{TURB} \cong 0.001\ m^2/s$ (really small mixing)

$= 10\ cm^2/s$ (Turbulent = Same for large & small clays)

LARGE $Z^1_{\frac{1}{2}} = \dfrac{0.69}{(7\times10^{-5}/10)} = 100{,}000\ cm = 1\ km$

SMALL $Z^2_{\frac{1}{2}} = \dfrac{0.69}{5\times10^{-7}/10} = 14\times10^6\ cm = 140\ km$ !!

Holy cow!

So you might barely see an effect for large clay in the open ocean ($H \gtrsim 1\ km$) but NONE in a settling tank)